Computation of Thermal Convection with a Large Temperature Difference(Solutions of the Navier-Stokes Equations)

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Computation of Thermal Convection with a Large Temperature Difference

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A new method is developed to compute a flow with a large temperature difference. The compressible Navier-Stokes equations are solved based on the approximation that the pressure difference is small. This method is coupled with a new higher-order upwind scheme. Two-dimensional flows in a channel with partially heated wall and a flow past a heated circular cylinder are investigated, and this method has been proved to be very robust and general.

Introduction

Thermal convection is a very important flow phenomenon in engineering applications. Many computations are done in this field but most of them are based on the Boussinesq approximation, which is only valid for relatively small temperature differences. If the temperature difference is more than $100^{\circ}C$, the effect of compressibility is not negligible, whereas it is not necessary to solve the full Navier-Stokes equations for this kind of problems in the subsonic range.

In this paper, a new method is developed to simulate a flow having large temperature differences. The compressible Navier-Stokes equations are solved based on the approximation only that the pressure difference is small in the flow field; this means that there appears no shock waves. This method is an extension of the MAC method, widely used for the computation of incompressible flows, to compressible flows with small pressure differences. This latter case occurs when the density difference comes only from the temperature difference. This method is coupled with a new higher-order upwind scheme which has been proved to be very effective for the computation of high-Reynolds-number flow[1].

Two-dimensional flows in a channel with partially heated wall are solved by this method. A flow past a heated circular cylinder is also investigated.

Computational Method

The incompressible Navier-Stokes equations are expressed as follows:

$$\mathbf{w} + \text{grad } p = \mathbf{F} , \quad \mathbf{w} = \frac{\partial \mathbf{v}}{\partial t} \tag{1}$$

$$\text{div } \mathbf{w} = 0 , \tag{2}$$

where the vector $\mathbf{F}$ is

$$\mathbf{F} = - (\mathbf{v}.\text{grad})\mathbf{v} + \frac{1}{Re} \mathbf{v} \tag{3}$$

The MAC method is essentially based on the Helmholtz decomposition[1,2] of the vector **F**. This decomposition can be completed by the following iteration

$$\mathbf{w}^n = \mathbf{F} - \mathrm{grad}\; p^n \tag{4}$$

$$p^{n+1} = p^n - \varepsilon\, \mathrm{div}\, \mathbf{w}^n \;. \tag{5}$$

The present method is based on this type of decomposition of vector **F**, where the second equation is modified because the acceleration **w** is no longer divergent free. This modification can be done as follows: the basic equations are the compressible Navier-Stokes equations:

$$\frac{1}{\rho}\frac{D\rho}{Dt} = -\frac{\partial u_j}{\partial x_j} \tag{6}$$

$$\frac{Du_i}{Dt} = X_i - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\right) \tag{7}$$

$$\frac{Dc_vT}{Dt} = \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(k\frac{\partial T}{\partial x_j}\right) - \frac{p}{\rho}\frac{\partial u_j}{\partial x_j} \tag{8}$$

where the energy equation (8) is expressed by using the temperature as a dependent variable; the dissipation function term is neglected because it plays no role so far as the flow is not supersonic. For simplicity, the viscosity μ and thermal diffusion coefficient k are assumed to be constant. The fluid is considered to be a perfect gas, that is

$$\rho = \frac{p}{RT} \tag{9}$$

If the pressure is nearly constant, then the change of the density can be expressed by that of the temperature:

$$\frac{1}{\rho}\frac{D}{Dt} = -\frac{1}{T}\frac{DT}{Dt} \tag{10}$$

Then the equation of continuity (6) becomes

$$\frac{\partial u_j}{\partial x_j} = \frac{1}{T}\frac{DT}{Dt} \tag{11}$$

Putting this into Eq.(8), we obtain

$$\frac{DT}{Dt} = \frac{k}{\rho(c_v+R)}\frac{\partial^2 T}{\partial x_j^2} \tag{12}$$

Therefore, the continuity equation (6) can be written as

$$\frac{\partial u_j}{\partial x_j} = \frac{k}{(c_v+R)\rho}\frac{1}{T}\frac{\partial^2 T}{\partial x_j^2} \tag{13}$$

This suggests that instead of the Helmholtz decomposition (4) and (5) the following decomposition works for the flow with high temperature difference:

$$\mathbf{w}^n = \mathbf{F} - \frac{1}{\rho n}\,\mathrm{grad}\, p^n, \quad \rho^n = \frac{p^n}{RT} \tag{14}$$

$$p^{n+1} = p^n - \varepsilon(\mathrm{div}\,\mathbf{w}^n - \frac{\partial}{\partial t}(\frac{k}{\rho(c_v+R)}\frac{1}{T}\frac{\partial^2 T}{\partial x_j^2})) \tag{15}$$

A small modification is needed to prevent the accumulation of the error in **w**; (the same technique was used in the Ref.2). This scheme is essentially the same as the MAC method but the effect of compressibility due to temperature is correctly evaluated. This method is coupled with a new higher-order upwind scheme (see Appendix) which makes the computation very stable. The other derivatives are approximated by central differences and the first-order Euler implicit scheme is used for the time integration.

Results

Two-dimensional flow problems are solved by using the above method. The first problem we consider is the flow in a channel where a part of the wall is heated at 1000 degrees C and the temperature of the inflow is 15 degrees; the Reynolds number is 1839. Figure 1 shows the time development of the flow. The effect of the compressibility is easily seen from the velocity vector plots; the total flux increases following the heat source.

The second problem is the flow past a circular cylinder heated at 1000 degrees C and the temperature of the inflow is 15 degrees. The Reynolds number is 2000. The temperature distribution and the velocity fields are shown in Fig.2.

Conclusion

The method developed in this paper has been found to be very robust and general for flows with large temperature difference. The only restriction is that the flow velocity be in the subsonic range.

Appendix

The non-linear term $f\frac{\partial u}{\partial x}$ is approximated by the following third order upwind scheme:

$$\left(f\frac{\partial u}{\partial x}\right)_{i,j} = f_{i,j}(-u_{i+2,j} + 8(u_{i+1,j} - u_{i-1,j}) + u_{i-2,j})/12h + |f_{i,j}|\,(u_{i+2,j} - 4u_{i+1,j} + 6u_{i,j} - 4u_{i-1,j} + u_{i-2,j})/4h$$

Reference

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2) H.Takami and K.Kuwahara: Numerical Study of Three-Dimensional Flow within a Cubic Cavity. J.Phys.Soc.Japan,Vol.37,No.6 (1974) 1695.

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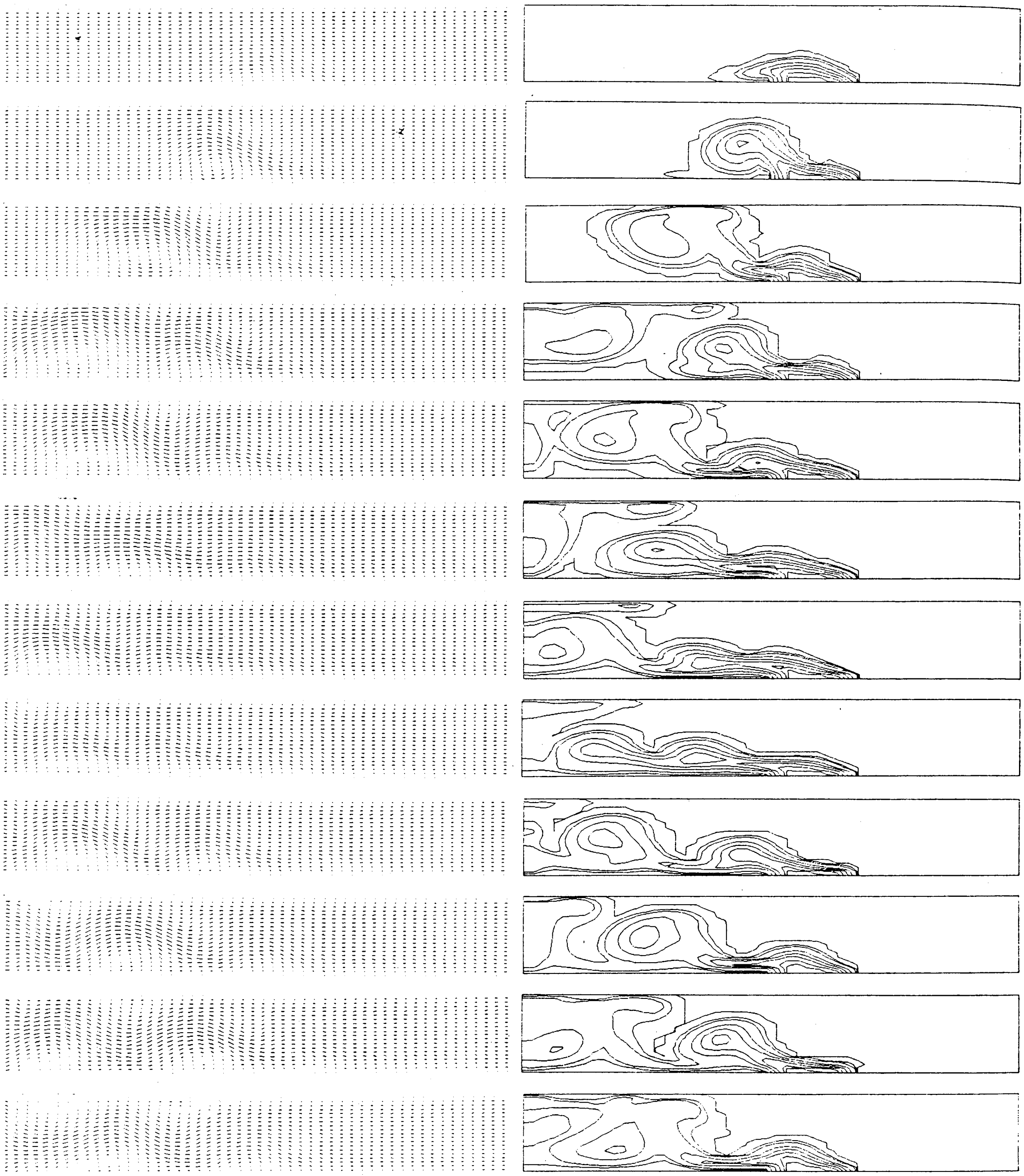

Fig.1 The time development of the flow in a channel.
The velocity vectors and temperature contours.
t=5,10,$\cdots$,60.

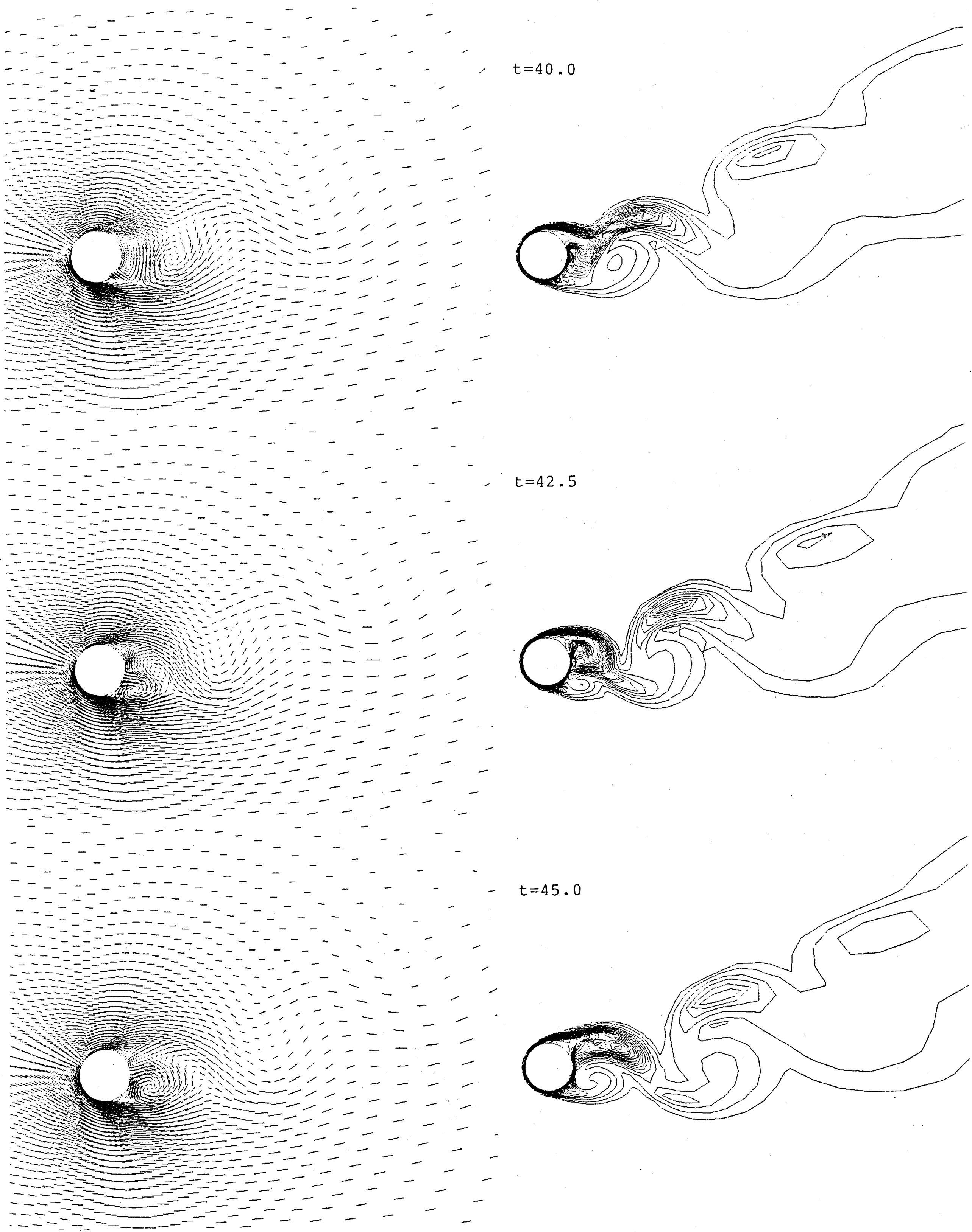

Fig.2 Flow past a heated circular cylinder of 1000°; the temperature of the inflow is 15°.